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SUSY $SU(5)$ with singlet plus adjoint matter and A_4 family symmetry

Iain K. Cooper, Stephen F. King, Christoph Luhn *

School of Physics and Astronomy, University of Southampton, Southampton, SO17 1BJ, UK

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ABSTRACT

We propose a supersymmetric (SUSY) $SU(5)$ Grand Unified Theory (GUT) including a single right-handed neutrino singlet and an adjoint matter representation below the GUT scale and extend this model to include an A_4 family symmetry and a gauged anomaly-free Abelian group. In our approach hierarchical neutrino masses result from a combined type I and type III seesaw mechanism, and the A_4 symmetry leads to tri-bimaximal mixing which arises indirectly. The mixing between the single right-handed neutrino and the matter in the adjoint is forbidden by excluding an adjoint Higgs, leading to a diagonal heavy Majorana sector as required by constrained sequential dominance. The model also reproduces a realistic description of quark and charged lepton masses and quark mixings, including the Georgi–Jarlskog relations and the leptonic mixing sum rules $s = r\cos\delta$ and $a = -r^2/4$ with $r = \theta_C/3$.

1. Introduction

There has been much evidence to suggest that the Standard Model (SM) is not a complete description of particle physics and needs extending. Arguably one of the most important pieces of experimental evidence for physics beyond the SM is the measurement of small but non-zero neutrino mass, leading to theories of neutrino mass and mixing. In the seesaw mechanism, a natural explanation for such tiny neutrino masses is provided by the exchange of a heavy particle leading to Majorana neutrino masses suppressed by the large mass of the exchanged particle. The heavy particle in the seesaw mechanism must be a colour singlet but can be an electroweak singlet fermion with zero hypercharge, an electroweak triplet Higgs scalar with two units of hypercharge, or an electroweak triplet fermion with zero hypercharge, corresponding to the type I [1], type II [2], or type III [3] seesaw mechanisms, respectively. In this Letter we shall combine the seesaw mechanism of types I and III in a new way to yield a hierarchical spectrum of neutrino masses.

However the seesaw mechanism is not by itself enough to account for the discovery that, in contrast with the smallness of the quark mixing angles, two out of the three leptonic mixing angles are large. This unexpected observation calls for a deeper theoretical understanding of the physics underlying the structure of fermion masses and mixings. It is well known that solar and atmospheric data are consistent with a simple form of the leptonic mixing matrix U, known as tri-bimaximal (TB) mixing [4]:

$$U_{TB} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}. \tag{1}$$

The simple form of this matrix can be interpreted as a clue that points towards some underlying family symmetry G_f, related to particular transformations which leave the mass matrix diagonalised by U_{TB} invariant. There has been much recent work based on this idea that the postulated TB symmetry can arise from a family symmetry [5–25]. The approaches taken in the literature may be separated into two distinct classes, distinguished by the breaking of the family symmetry [26]: *direct* models, based on A_4, S_4, or larger groups containing these as subgroups [27,28], have part of the family symmetry preserved at low energies and this forms some or all of the neutrino flavour symmetry; *indirect* models, usually based on $\Delta(3n^2)$ or $\Delta(6n^2)$ [29], have entirely broken family symmetries (in the neutrino sector), and the neutrino flavour symmetry appears accidentally.

Finding an explanation for the distinctly different mixing patterns of leptons as compared to quarks is even more important in the context of Grand Unified Theories (GUTs) [30,31], where the fermionic matter is unified at high energies into either a single representation, as in $SO(10)$ [32] or E_6 [33], or into two representations, as in $SU(5)$ [34] or $SU(4)_{PS} \times SU(2)_L \times SU(2)_R$ [35]. The minimal GUT [34] is based on the Lie group $SU(5)$, where one family of right-handed down quarks and left-handed leptons are unified in a $\bar{\mathbf{5}}$ and the rest of the family are in a $\mathbf{10}$. Three copies of each of these representations then constitute the full fermionic matter content of minimal $SU(5)$. It is well known that gauge coupling unification fails in this regime, however if promoted to a

* Corresponding author.
E-mail addresses: ikc1g08@soton.ac.uk (I.K. Cooper), sfk@hep.phys.soton.ac.uk (S.F. King), christoph.luhn@soton.ac.uk (C. Luhn).

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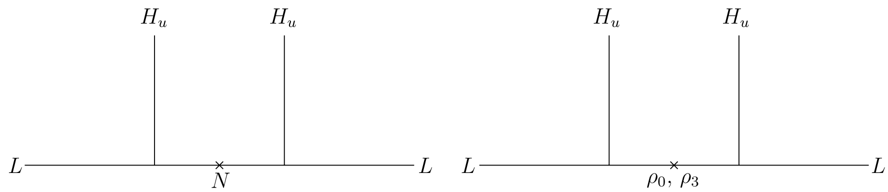

Fig. 1. Schematic diagrams of the type I (left) and combined type I + type III (right) seesaw mechanisms present in the model. The seesaw messenger states are N and the ρ_0, ρ_3 components of $\psi_{\mathbf{24}}$. L is the $SU(2)_L$ doublet contained in the $\bar{\mathbf{5}}$ of $SU(5)$.

supersymmetric (SUSY) GUT [36] with two Higgs multiplets, $H_{\mathbf{5}}$ and $H_{\bar{\mathbf{5}}}$, then unification occurs at a scale of roughly 2×10^{16} GeV [37].

It is clear that neutrino masses are zero at the renormalisable level in minimal (SUSY) $SU(5)$ as in the minimal SM. However, in both the SM and $SU(5)$, as pointed out by Weinberg [38], one may invoke a non-renormalisable dimension-5 operator at or above the GUT scale to generate neutrino masses. Such an operator at the GUT scale may be sufficient to describe the solar neutrino mass scale, but not the atmospheric neutrino mass scale. In order for neutrinos to obtain mass consistent with atmospheric mixing in a (SUSY) $SU(5)$ model, the seesaw mechanism is a very attractive possibility, however this requires some extra matter or Higgs to be added below the GUT scale. The choice of additional matter or Higgs is very *ad hoc* since the $SU(5)$ theory does not specify the nature of this extra matter and only requires that it be anomaly-free. A popular choice is to add three right-handed neutrinos which arise from singlet $SU(5)$ representations. However the number of singlets is not predicted in $SU(5)$, and it is possible to add just a single right-handed neutrino to describe the atmospheric mass scale [39]. In order to describe both atmospheric and solar neutrino masses with two large mixing angles using the type I seesaw mechanism two right-handed neutrinos are sufficient [40]. However, within $SU(5)$ GUTs, there are other possibilities.

It has been pointed out that, in (SUSY) $SU(5)$ GUTs, non-fundamental matter multiplets have decompositions which include both fermion singlets and fermion triplets suitable for the types I and III seesaw mechanism, the smallest such example being the adjoint **24** representation [41–43]. The decomposition of a matter **24** under the SM gauge group $SU(3)_c \times SU(2)_L \times U(1)_Y$ involves an $SU(2)_L$ singlet $\rho_0 = (\mathbf{1}, \mathbf{1})_0$ as well as a triplet $\rho_3 = (\mathbf{1}, \mathbf{3})_0$, thus leading to a combination of a type I seesaw with a type III seesaw [3]. However, assuming the simplest Higgs sector, the ρ_0 and ρ_3 are constrained by $SU(5)$ to give equal contributions to the neutrino mass matrix, up to an overall constant, resulting in a rank one neutrino mass matrix and only one non-zero neutrino mass. This problem may be addressed by allowing additional couplings to a Higgs **45** [43], but here we shall consider a different possibility.

In this Letter we consider a SUSY $SU(5)$ GUT with a single right-handed neutrino singlet superfield N plus one adjoint matter superfield $\psi_{\mathbf{24}}$ below the GUT scale. The model combines a type I seesaw mechanism from the single right-handed neutrino N below the GUT scale [39] with a type I plus type III seesaw mechanism from the ρ_0 and ρ_3 components contained in the single adjoint matter superfield $\psi_{\mathbf{24}}$ below the GUT scale [43]. The seesaw mechanism in our model therefore results from three distinct diagrams as shown in Fig. 1. In order to describe TB mixing we also include an A_4 family symmetry, plus an anomaly-free gauged $U(1)$ symmetry. Instead of using an adjoint Higgs representation $H_{\mathbf{24}}$ to spontaneously break $SU(5)$ to the SM gauge group, we shall assume implicitly that the GUT group is broken by geometrical effects in extra dimensions. However the theory here is formulated in four dimensions and we simply assume that it could subsequently be uplifted to a higher-dimensional setting (as in, for example, [44]). The absence of $H_{\mathbf{24}}$ is crucial in forbidding the mixing between the right-handed neutrino N and $\psi_{\mathbf{24}}$, leading to no mass mixing between N and ρ_0 and hence a diagonal heavy Majorana sector as required by constrained sequential dominance (CSD) [45]. The flavon vacuum alignments arise from the elegant D-term mechanism [46]. The model also reproduces a realistic description of quark and charged lepton masses and quark mixings, including the Georgi–Jarlskog relations [47].

The remainder of this Letter is organised as follows. In Section 2 we introduce the SUSY $SU(5)$ model with singlet plus adjoint matter below the GUT scale. Section 3 describes the full version of the model, including an A_4 family symmetry, a gauged $U(1)$ symmetry plus two discrete Z_2 symmetries, and lists the operators allowed by these symmetries, resulting in the mass matrices for quarks, charged leptons and neutrinos. We conclude in Section 4.

2. SUSY $SU(5)$ with singlet and adjoint matter

In this section we consider a SUSY $SU(5)$ GUT with one single right-handed neutrino arising from a singlet representation N below the GUT scale plus one extra adjoint matter representation $\psi_{\mathbf{24}}$ with mass also below the GUT scale. The matter contained in the $\psi_{\mathbf{24}}$ is degenerate thus avoiding problems with gauge coupling unification. The model represents a new way to achieve a hierarchical neutrino mass spectrum arising from a type I plus type III seesaw mechanism, as we now discuss.

The superpotential describing the neutrino sector takes the form

$$W = c_i F_i \psi_{\mathbf{24}} H_{\mathbf{5}} + p_i F_i N H_{\mathbf{5}} + \frac{1}{2} m_N N N + \frac{1}{2} m \operatorname{Tr}\left(\psi_{\mathbf{24}}^2\right), \tag{2}$$

with the F_i denoting the three families of $\bar{\mathbf{5}}$s. N is a single right-handed Majorana neutrino superfield and $\psi_{\mathbf{24}}$ the additional adjoint matter superfield. The seesaw diagrams illustrated in Fig. 1 then yield the light neutrino mass matrix

$$M_\nu^{ij} = c_i c_j v_u^2 \left(\frac{1}{4 m_{\rho_3}} + \frac{3}{20 m_{\rho_0}} \right) + \frac{p_i p_j}{m_N} v_u^2 . \tag{3}$$

Here v_u is the vacuum expectation value (VEV) of the Minimal Supersymmetric Standard Model (MSSM) Higgs field H_u which corresponds to the $SU(2)_L$ doublet within the $SU(5)$ Higgs $H_{\mathbf{5}}$. The numerical factors of the two terms in parentheses are obtained by writing $\psi_{\mathbf{24}} = \tilde{\rho}_a T^a$, where $\tilde{\rho}_a$ are the 24 components of the $\psi_{\mathbf{24}}$ and T^a are the 24 appropriately normalised generators of $SU(5)$ [30]. As can be seen from Eq. (2), the Majorana masses for the seesaw messengers ρ_0 and ρ_3 are identical, i.e. $m_{\rho_0} = m_{\rho_3} = m$, while N has an independent mass m_N. Note that we have not introduced an adjoint Higgs $H_{\mathbf{24}}$ which would break the degeneracy of the components in the $\psi_{\mathbf{24}}$ and, more importantly, allow a mixing term $N \psi_{\mathbf{24}} H_{\mathbf{24}}$ leading to a mass mixing between N and ρ_0. Note also that c_i and p_i are independent dimensionless coefficients (where i and j are family indices); this independence is crucial to

Table 1
Matter, Higgs and flavon chiral superfields in the model. The $U(1)$ charge q_1 can take any value which prevents φ_1 from significantly interacting with the other fields of the model, for instance $q_1 = -\frac{126}{24}$ as discussed below.

Field	$\psi_{\mathbf{24}}$	N	F	T_1	T_2	T_3	$H_{\mathbf{5}}$	$H_{\bar{\mathbf{5}}}$	$H_{\overline{\mathbf{45}}}$	φ_{123}	φ_{23}	φ_3	ξ	ξ'	φ_1
$SU(5)$	**24**	**1**	$\bar{\mathbf{5}}$	**10**	**10**	**10**	**5**	$\bar{\mathbf{5}}$	$\overline{\mathbf{45}}$	**1**	**1**	**1**	**1**	**1**	**1**
A_4	**1**	**1**	**3**	**1**	**1**	**1**	**1**	**1**	**1**	**3**	**3**	**3**	**1**	**1**	**3**
$U(1)_R$	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0
$U(1)$	−1	2	0	4	1	0	0	0	2	1	−2	0	−1	−4	q_1
Z_2^1	−	−	+	+	+	+	+	−	−	−	−	−	+	−	+
Z_2^2	+	+	+	+	+	−	+	+	+	+	+	−	+	+	+

obtaining a rank two mass matrix and thus two non-zero neutrino masses.

As c_i and p_i are uncorrelated parameters, Eq. (3) does not in general conform to the TB structure of the neutrino mass matrix. It is the aim of this Letter to obtain TB neutrino mixing as a consequence of a discrete family symmetry in this type of model. To this end, in the next section, we augment the adjoint SUSY $SU(5)$ model with the tetrahedral family symmetry A_4.

3. SUSY $A_4 \times SU(5)$ with singlet and adjoint matter

In this section we uplift the model in Eq. (2) to include a tetrahedral family symmetry. We work in the basis of [15] in which two A_4 triplets $a = (a_1, a_2, a_3)^T$ and $b = (b_1, b_2, b_3)^T$ give a singlet through the combination $a_1 b_1 + a_2 b_2 + a_3 b_3$. The basic idea is to unify the three families of $\bar{\mathbf{5}}$s into an A_4 triplet $F \sim \mathbf{3}$, and in order for Eq. (2) to remain invariant, to introduce extra A_4 triplets called flavons φ_i to break the A_4 symmetry and generate the Yukawa couplings.

Table 1 shows the chiral superfields present in the model. As mentioned above, the three $\bar{\mathbf{5}}$s of $SU(5)$ are embedded in a triplet of A_4, while the three **10**s are singlets. The $\psi_{\mathbf{24}}$ is an A_4 singlet as is the right-handed neutrino N. We have fundamental Higgs fields $H_{\mathbf{5}}$ and $H_{\bar{\mathbf{5}}}$; introducing another Higgs in the $\overline{\mathbf{45}}$ representation, $H_{\overline{\mathbf{45}}}$, enables the implementation of the Georgi–Jarlskog mechanism [47] to obtain the well-known GUT scale mass relations $m_e \sim \frac{m_d}{3}$, $m_\mu \sim 3 m_s$ and $m_\tau \sim m_b$. These give phenomenologically successful predictions of down quark and charged lepton masses when evolved down to the electroweak scale.

The $U(1)_R$ represents an R-symmetry. Its Z_2 subgroup gives rise to the standard R-parity which forbids unwanted operators contributing to proton decay and keeps the lightest SUSY particle a good candidate for cold dark matter. Moreover, $U(1)_R$ is essential in forbidding F-term contributions to the flavon superpotential which otherwise could dominate the relevant D-term operators used for obtaining the desired vacuum alignment (see below and the discussion in [46]). The $U(1)$ and the two Z_2 symmetries constrain the structure of the Yukawa matrices in the quark and charged lepton sectors. The standard MSSM μ-term[1] $\mu H_u H_d$ is forbidden by the first of the Z_2 symmetries as well as by $U(1)_R$, allowing for a natural solution to the μ-problem of the MSSM using a GUT singlet from the hidden sector of Supergravity theories [48].

The flavon fields φ_i, ξ and ξ' break the A_4 symmetry and constrain the form of the lepton and down quark Yukawa matrices. The vacuum alignments of the triplet flavon VEVs that we assume in this model are displayed in Table 2. They are achieved using the D-term vacuum alignment mechanism discussed recently in [46]. This mechanism is ideally suited for models such as this in which

Table 2
The vacuum alignments of the triplet flavons used in the model. Without loss of generality, the alignments are given without phases; the relative sign between $\langle\varphi_{23}\rangle_2$ and $\langle\varphi_{23}\rangle_3$ is relevant, though the actual position of the minus sign is mere convention.

Flavon VEV	VEV alignment
$\langle\varphi_1\rangle$	$(1, 0, 0)^T$
$\langle\varphi_3\rangle$	$(0, 0, 1)^T$
$\langle\varphi_{23}\rangle$	$\frac{1}{\sqrt{2}}(0, 1, -1)^T$
$\langle\varphi_{123}\rangle$	$\frac{1}{\sqrt{3}}(1, 1, 1)^T$

the flavons are used to generate the neutrino flavour symmetry as an indirect result of the A_4 symmetry as discussed in [26]. Moreover, the D-term vacuum alignment mechanism does not involve the introduction of extra "driving fields" in the superpotential and does not impose any restrictions on the model other than the requirement that higher order terms in the flavon potential do not spoil the vacuum alignment arising from the D-terms. This has been demonstrated to arise in a fairly generic way in [46] providing that the model also respects a $U(1)_R$ symmetry and involves no superfields with $R = 2$ which, like driving fields, could appear linearly in the superpotential and lead to large terms in the flavon potential. The present model involves only fields with $R = 0, 1$ and so the D-term flavon potential will not receive large corrections from the superpotential. Since the D-term vacuum alignment mechanism is generic and does not provide any other restrictions on the model than those stated, in this Letter we shall simply assume that this mechanism is in operation, leading to the stated alignments for φ_{123}, φ_{23}, φ_3, φ_1.

In order to avoid the massless Goldstone boson associated with the spontaneously broken $U(1)$ symmetry, we assume it to be gauged.[2] In addition to the particle content specified in Table 1 we must then introduce extra matter to cancel the respective gauge anomalies. The cubic $SU(5)$ anomaly requires the introduction of a Higgs field $H_{\mathbf{45}}$ whose $U(1)$ charge is determined by the mixed $SU(5) - SU(5) - U(1)$ anomaly to be $q(H_{\mathbf{45}}) = -\frac{53}{24}$. Finally the cubic $U(1)$ anomaly can be removed in many ways; for example, choosing $q_1 = -\frac{126}{24}$ we can add three extra $A_4 \times SU(5)$ singlets with $U(1)$ charges $\frac{5}{24}$, $\frac{25}{24}$, $\frac{51}{24}$. Assuming that $H_{\mathbf{45}}$ has the same Z_2 charges as $H_{\overline{\mathbf{45}}}$ while the three extra $A_4 \times SU(5)$ singlets are neutral under both Z_2 symmetries, we have checked that these additional fields lead to only negligible contributions to the fermion mass matrices discussed below, provided they get VEVs of order $\epsilon\Lambda$ or smaller, see Eq. (8).

1 Where H_u is the SM doublet of $H_{\mathbf{5}}$; and H_d is a linear combination of the SM doublets in $H_{\bar{\mathbf{5}}}$ and $H_{\overline{\mathbf{45}}}$.

2 If it were not gauged, Goldstone boson masses could arise from explicit $U(1)$ breaking in the hidden sector which could generate soft SUSY breaking terms involving only flavon fields where such terms explicitly violate the $U(1)$. However such terms could jeopardise the D-term alignment mechanism so here we prefer to gauge the $U(1)$ to avoid any potential problems.

3.1. Allowed terms

The neutrino sector is composed of Dirac and Majorana mass terms which take the form in the superpotential:

$$W_\nu = \frac{\varphi_{123}}{\Lambda} cF\psi_{\mathbf{24}}H_{\mathbf{5}} + \frac{\varphi_{23}}{\Lambda} pFNH_{\mathbf{5}} + \frac{\varphi_{23}^2}{2\Lambda} y_N NN + \frac{\xi^4}{2\Lambda^3} y'_N NN + \frac{\varphi_{123}^2}{2\Lambda} y \operatorname{Tr}(\psi_{\mathbf{24}}^2), \tag{4}$$

with Λ a heavy mass scale and c, p, y_N, y'_N, y dimensionless coupling constants. When the flavons get their VEVs the superpotential in Eq. (4) reproduces that in Eq. (2) but with constrained couplings c_i and p_i leading to TB mixing.

The superpotential terms of the down quark and charged lepton sector are given as follows

$$W_d \sim \frac{\varphi_{23}\xi^2}{\Lambda_d^3} T_1 F H_{\overline{\mathbf{5}}} + \frac{\varphi_{123}\xi^2}{\Lambda_d^3} T_2 F H_{\overline{\mathbf{5}}} + \frac{\varphi_{23}\xi}{\Lambda_d^2} T_2 F H_{\overline{\mathbf{45}}} + \frac{\varphi_3}{\Lambda_d} T_3 F H_{\overline{\mathbf{5}}}, \tag{5}$$

where Λ_d is the relevant messenger mass. The flavon ξ plays a role similar to a Froggatt–Nielsen field [49], except that it is not the sole contributor to the generated mass hierarchy, here combined as it is with the triplet flavons.

Finally the up quark sector Yukawa superpotential terms take the form

$$W_u \sim \frac{(\xi')^2}{\Lambda_u^2} T_1 T_1 H_{\mathbf{5}} + \left(\frac{\varphi_{23}^2\xi}{\Lambda_u^3} + \frac{\xi^5}{\Lambda_u^5}\right)(T_1T_2 + T_2T_1)H_{\mathbf{5}} + \frac{\varphi_{23}\varphi_3\xi^2}{\Lambda_u^4}(T_1T_3 + T_3T_1)H_{\mathbf{5}} + \frac{\xi^2}{\Lambda_u^2} T_2T_2H_{\mathbf{5}} + \frac{\varphi_{123}\varphi_3\xi^2}{\Lambda_u^4}(T_2T_3 + T_3T_2)H_{\mathbf{5}} + T_3T_3H_{\mathbf{5}}. \tag{6}$$

It should be mentioned that the messenger mass in this sector, Λ_u, may in principle be different from that in the down quark sector. The field ξ' is introduced specifically to generate the T_1T_1 term to the required order.

3.2. Fermion mass matrices

After spontaneous breakdown of the A_4 family symmetry by the flavon VEVs, the superpotential terms of Eqs. (4), (5) and (6) predict mass matrices for the respective sectors. In the following, order one coefficients in the quark and charged lepton sectors are omitted (including flavon VEV normalisation factors). Regarding the scale of the flavon VEVs we define

$$\eta_i = \frac{\langle|\varphi_i|\rangle}{\Lambda}, \tag{7}$$

where $\varphi_i = \varphi_{123}$, φ_{23}, φ_3, ξ or ξ'. In order to get the hierarchical structure of the quark and charged lepton mass matrices we assume[3]

$$\eta_{123}, \eta_{23}, \eta_{\xi'} = \epsilon^2 \quad \text{and} \quad \eta_3, \eta_\xi = \epsilon, \tag{8}$$

where the numerical values for ϵ depend on the messenger scale of the relevant sector. We note that we have given the superpotential terms of the quark and charged lepton sectors up to and including $\mathcal{O}(\epsilon^5)$.

[3] It is possible to have a hierarchy in the flavon VEVs since the scales at which their mass terms are driven negative can vary [46].

In the Higgs sector, it is not the $H_{\mathbf{5}}$, $H_{\overline{\mathbf{5}}}$ or $H_{\overline{\mathbf{45}}}$ which get VEVs but their SM doublet components. These are the two MSSM doublets H_u (corresponding to $H_{\mathbf{5}}$) and H_d (corresponding to a linear combination of $H_{\overline{\mathbf{5}}}$ and $H_{\overline{\mathbf{45}}}$); they originate below the GUT scale and remain massless down to the electroweak scale. The non-MSSM states all acquire GUT scale masses, including the linear combination of $H_{\overline{\mathbf{5}}}$ and $H_{\overline{\mathbf{45}}}$ orthogonal to H_d. Electroweak symmetry is broken after the light MSSM doublets $H_{u,d}$ acquire VEVs $v_{u,d}$ and they then generate the fermion masses.

In the following all quark and charged lepton mass matrices are given in the L-R convention, i.e. the mass term for a field ψ is given in the order $\psi_L M_{LR} \psi_R$.

3.2.1. Neutrino sector

In our model the light neutrino masses arise from a combination of type I and type III seesaw. Due to the absence of a $H_{\mathbf{24}}$ the heavy seesaw messenger particles N and ρ_0 do not mix as can be seen from Eq. (4). Thus the 2×2 Majorana mass matrix of the heavy right-handed $SU(2)_L$ singlets is automatically diagonal. Furthermore, the seesaw messenger responsible for the type III contribution, ρ_3, cannot mix with N as they furnish different $SU(2)_L$ representations. In CSD the (approximate) diagonal nature of the seesaw particles is usually a necessary extra assumption which often lacks a fundamental explanation. In our adjoint model, however, it is directly built into the theory by not including $H_{\mathbf{24}}$. Therefore our model represents a very natural realisation of CSD.

In the Dirac neutrino sector of Eq. (4), the spontaneous breaking of the A_4 family symmetry by the flavon VEVs $\langle\varphi_{123}\rangle$ and $\langle\varphi_{23}\rangle$ gives

$$\mathcal{L}_\nu = \frac{c\eta_{123}v_u}{\sqrt{3}}(\nu_e + \nu_\mu + \nu_\tau)\left(\frac{\rho_3^0}{2} - \sqrt{\frac{3}{20}}\rho_0\right) - \frac{p\eta_{23}v_u}{\sqrt{2}}(\nu_\mu - \nu_\tau)N + \text{h.c.}, \tag{9}$$

where the numerical factors of ρ_3^0 and ρ_0 are determined from the normalised $SU(5)$ generators in the adjoint representation [30]. Upon application of the seesaw formula of Eq. (3) we find the effective left-handed Majorana neutrino mass matrix

$$M_\nu = \frac{2c^2v_u^2}{15y\Lambda}\begin{pmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{pmatrix} + \frac{p^2v_u^2}{2(y_N + y'_N\eta_\xi^4/\eta_{23}^2)\Lambda}\begin{pmatrix}0 & 0 & 0\\ 0 & 1 & -1\\ 0 & -1 & 1\end{pmatrix}. \tag{10}$$

Since any matrix diagonalisable by Eq. (1) may be written as[4] $m_1\varphi'_1(\varphi'_1)^T/|\varphi'_1|^2 + m_2\varphi_{123}(\varphi_{123})^T/|\varphi_{123}|^2 + m_3\varphi_{23}(\varphi_{23})^T/|\varphi_{23}|^2$ [26], we may readily read off the masses and state that

$$M_\nu^{\text{diag}} = \begin{pmatrix}0 & 0 & 0\\ 0 & m_2 & 0\\ 0 & 0 & m_3\end{pmatrix}, \quad \text{with}$$

$$m_2 = \frac{2c^2v_u^2}{5y\Lambda}, \qquad m_3 = \frac{p^2v_u^2}{(y_N + y'_N\eta_\xi^4/\eta_{23}^2)\Lambda}. \tag{11}$$

Hence the model predicts one massless left-handed neutrino and thus a hierarchical neutrino mass spectrum.

[4] $\varphi'_1 \propto \frac{1}{\sqrt{6}}(-2, 1, 1)^T$.

3.2.2. Down quark and charged lepton sector

In the down quark and charged lepton sector, the superpotential of Eq. (5) predicts a mass matrix of the form (with messenger mass Λ_d in η_i)

$$\begin{pmatrix} 0 & \eta_{23}\eta_\xi^2 & -\eta_{23}\eta_\xi^2 \\ \eta_{123}\eta_\xi^2 & \eta_{123}\eta_\xi^2 + k_f\eta_{23}\eta_\xi & \eta_{123}\eta_\xi^2 - k_f\eta_{23}\eta_\xi \\ 0 & 0 & \eta_3 \end{pmatrix} v_d, \tag{12}$$

where k_f is the Georgi–Jarlskog factor (in the case that $f = e$, the mass matrix must also be transposed):

$$k_f = \begin{cases} 1 & \text{for } f = d, \\ -3 & \text{for } f = e. \end{cases}$$

Inserting the ϵ suppressions of the flavon VEVs from Eq. (8) the down quark mass matrix becomes

$$M_d \sim \begin{pmatrix} 0 & \epsilon^3 & -\epsilon^3 \\ \epsilon^3 & \epsilon^2 & -\epsilon^2 \\ 0 & 0 & 1 \end{pmatrix} \epsilon v_d, \tag{13}$$

whilst the charged lepton mass matrix reads

$$M_e \sim \begin{pmatrix} 0 & \epsilon^3 & 0 \\ \epsilon^3 & -3\epsilon^2 & 0 \\ -\epsilon^3 & 3\epsilon^2 & 1 \end{pmatrix} \epsilon v_d. \tag{14}$$

Here we assume the numerical value $\epsilon \sim 0.15$. Upon diagonalisation, these give mass ratios of $\epsilon^4 : \epsilon^2 : 1$ for the down quarks and $\frac{\epsilon^4}{3} : 3\epsilon^2 : 1$ for the charged leptons. These ratios are in good agreement with quark and lepton data and also predict GUT scale mass relations of $m_e \sim \frac{m_d}{3}$, $m_\mu \sim 3m_s$ and $m_\tau \sim m_b$ as desired. In the low quark angle approximation, left-handed down quark mixing angles $\theta_{12}^d \sim \epsilon$, $\theta_{13}^d \sim \epsilon^3$ and $\theta_{23}^d \sim \epsilon^2$ are also predicted in agreement with data (assuming an approximately diagonal up sector which we obtain in the next section). The corresponding charged lepton mixing angles are $\theta_{12}^e \sim \frac{\epsilon}{3}$, $\theta_{13}^e \sim 0$ and $\theta_{23}^e \sim 0$.

The Pontecorvo–Maki–Nakagawa–Sakata (PMNS) matrix is not of exact TB form but receives small corrections from charged lepton mixing. In particular, the reactor angle deviates from zero by $\theta_{13} \sim \frac{1}{\sqrt{2}}\frac{\epsilon}{3}$ [50]. Furthermore, since $\theta_{13}^e \sim \theta_{23}^e \sim 0$, two sum rules for lepton mixing are respected [50,51]. Expressed in terms of the (r)eactor, (s)olar and (a)tmospheric deviation parameters defined as $\sin\theta_{13} = \frac{r}{\sqrt{2}}$, $\sin\theta_{12} = \frac{1}{\sqrt{3}}(1+s)$, $\sin\theta_{23} = \frac{1}{\sqrt{2}}(1+a)$ [52], the sum rules read $s = r\cos\delta$ and $a = -r^2/4$ [18], with δ being the leptonic Dirac CP phase.

3.2.3. Up quark sector

Eq. (6) may be expanded after A_4 symmetry breaking and is responsible for up quark masses:

$$\begin{pmatrix} \eta_{\xi'}^2 & \eta_{23}^2\eta_\xi + \eta_\xi^5 & -\eta_{23}\eta_3\eta_\xi^2 \\ \eta_{23}^2\eta_\xi + \eta_\xi^5 & \eta_\xi^2 & \eta_{123}\eta_3\eta_\xi^2 \\ -\eta_{23}\eta_3\eta_\xi^2 & \eta_{123}\eta_3\eta_\xi^2 & 1 \end{pmatrix} v_u. \tag{15}$$

Taking the VEV hierarchy as in Eq. (8), but now adopting the messenger scale $\Lambda_u \approx 3\Lambda_d$, we obtain a mass matrix with an expansion parameter $\bar{\epsilon} \sim 0.05$,

$$M_u \sim \begin{pmatrix} \bar\epsilon^4 & \bar\epsilon^5 & -\bar\epsilon^5 \\ \bar\epsilon^5 & \bar\epsilon^2 & \bar\epsilon^5 \\ -\bar\epsilon^5 & \bar\epsilon^5 & 1 \end{pmatrix} v_u \tag{16}$$

and an up quark mass hierarchy $\bar\epsilon^4 : \bar\epsilon^2 : 1$. As the mass matrix of Eq. (16) is diagonal to a good approximation, the up quark mixing is negligible. An important consequence of this observation is that the Cabibbo–Kobayashi–Maskawa (CKM) mixing arises predominantly from the down quark sector, with the Cabibbo angle being $\theta_C \sim \theta_{12}^d \sim \epsilon$.

4. Conclusions

In conclusion, minimal (SUSY) $SU(5)$ represents an attractive route to unification, but the Weinberg operator cannot account for neutrino mass and mixing, and the seesaw mechanisms all require extra matter or Higgs below the GUT scale. An appealing possibility, considered here, is to extend SUSY $SU(5)$ by assuming a single right-handed neutrino singlet and an adjoint matter representation below the GUT scale, including an A_4 family symmetry as well as a gauged anomaly-free $U(1)$. Hierarchical neutrino masses result from a combined type I and type III seesaw mechanism, and TB mixing arises indirectly from the A_4 family symmetry.

One attractive feature of this scheme is that the mixing between the single right-handed neutrino and the matter in the adjoint can be forbidden by not including the $H_{\mathbf{24}}$, leading to a diagonal heavy Majorana sector as required by CSD. The flavon vacuum alignments arise from the elegant SUSY D-term mechanism. The model also reproduces a realistic description of quark and charged lepton masses and quark mixings, including the Georgi–Jarlskog relations.

Corrections to TB mixing in the lepton sector come solely from the 1–2 mixing of the left-handed charged leptons, resulting in a PMNS matrix which is within the experimentally allowed limits. In particular the model respects the sum rules $s = r\cos\delta$ and $a = -r^2/4$ with $r = \theta_C/3$.

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